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ORDER AND REALIZABILITY OF IMPULSE RESPONSE FILTERS FOR ACCURATE IDENTIFICATION OF ACOUSTIC MULTI-PORTS FROM TRANSIENT CFD

Wolfgang Polifke, Alexander Gentemann
Lehrstuhl für Thermodynamik
Technische Universität München
D-85747 Garching, Germany
polifke@td.mw.tum.de

Abstract

For analysis of (thermo-)acoustic systems and in particular combustion instabilities, so-called *network models* are popular. Such models comprise multi-ports, which are represented mathematically by transfer matrices. Transfer matrices may be reconstructed from transient CFD simulation data with system identification tools, i.e. unit impulse response filters are determined with correlation analysis. The present study is concerned with the optimal choice of parameters for accurate transfer matrix reconstruction. Recommendations for the optimal choice of sample increment and sample length as well as filter order are formulated. Remarkably, it is found that in many cases the use of non-causal filters is advantageous. It is argued that this is a consequence of the fact that causal interrelationships imposed by the underlying laws of fluid mechanics are not always represented properly with the standard acoustic variables.

INTRODUCTION

For the purpose of analysis or control of complex (thermo-)acoustic systems, so-called *network models* are popular. With this approach, individual system elements are represented as multi-ports, described mathematically by their respective transfer matrices. In its fundamental form, a transfer matrix furnishes linear relationships between acoustic variables – e.g. fluctuations of pressure p' and velocity u' – at the different ports of an element. Combining the power of computational fluid dynamics (CFD) with the efficiency of network models, it has been proposed to use advanced tools from *system identification* to estimate or reconstruct transfer matrices from transient CFD data[1–4]. Time series data taken from a transient CFD calculation with low-amplitude forcing of pressure and/or velocity at the boundaries of the computational domain are regarded as (acoustic) signals received and responses emitted from the multi-port, respectively, and used to estimate auto- and cross-correlations of acoustic variables. From the correlations, unit impulse and frequency responses, i.e. the desired coefficients of the transfer

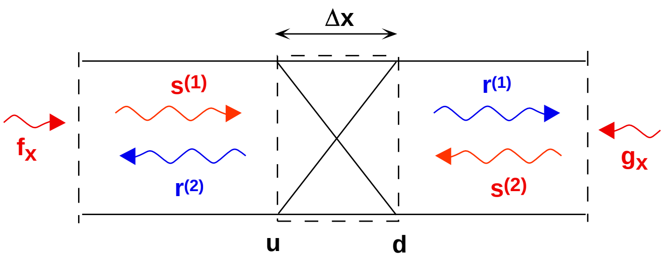

Figure 1: Schematic of the setup for correlation analysis with transient CFD: At the boundaries of the computational domain, transient forcing generates external waves f_x, g_x with desired spectral content. The element to be analyzed (”black box”) is part of the computational domain. If the scattering matrix formulation is used (see below), acoustic waves incident on the element are regarded as signals $s^{(1)}, s^{(2)}$, the reflected / transmitted waves are the responses $r^{(1)}, r^{(2)}$.

matrix[1], are computed.

The increment Δt of the time series, which need not be equal to the time step dt of the CFD simulation, and the number of coefficients of the impulse response vectors, i.e. the *order* of the response filters, are adjustable parameters in this procedure. Also, it is not obvious whether the system identification is best formulated in terms of primitive acoustic variables (u', p') or in terms of the Riemann invariants (f, g), and which terminals of the multi-port should be considered as signals (input) and responses (output), respectively. In the present paper, basic results of the theory of linear, time-invariant systems are combined with numerical analysis in order to derive guidelines for the selection of optimal parameter values, such that an optimal compromise between computational effort and numerical errors on the one hand, and most accurate estimation of the transfer matrix coefficients over an appropriate range of frequencies on the other hand, is achieved.

DISCRETE-TIME SYSTEMS AND CORRELATION ANALYSIS

In this section, fundamental concepts, methods and results concerning discrete-time systems and system identification are reviewed very briefly. Detailed presentations of this material can be found in many books[8–10], a short write-up has been made available on the internet[11].

Consider the values of acoustic variables up- and downstream of a network element (the ”black box”) as *signals* s and *responses* r, see fig. 1. A transient numerical simulation of compressible flow with external forcing provides time series $s_i = s(i\Delta t), i = 0, \ldots, N$ and similarly r_i. The *autocorrelation matrix* Γ of the signal $\mathbf{s}$ and the *cross-correlation* $\mathbf{c}$ between signal and response are approximated as follows:

$$c_i \approx \frac{1}{M} \sum_{l=L_{max}}^{N-L_{min}} s_{l-i} r_l \quad \text{and} \quad \Gamma_{ij} \approx \frac{1}{M} \sum_{l=L_{max}}^{N-L_{min}} s_{l-i} s_{l-j} \quad \text{for} \quad i, j = L_{min}, \ldots, L_{max}. \qquad (1)$$

How to choose the summation limits L_{min}, L_{max} is discussed below, the normalization constant $M \equiv N - L_{min} - L_{max} + 1$.

[1] Alternatively, it is possible to determine with these tools a (scalar) flame transfer function relating, e.g., fluctuations of velocity u' and heat release rate $\dot{Q}'$ to each other. In combination with suitable assumptions about the behavior of a flame, a complete flame transfer matrix can be derived for stability analysis of combustion systems[5–7].

For sufficiently small amplitudes of fluctuations, a transient CFD simulation approximates a discrete-time, real-valued, time-invariant linear system[8–11] (LTI system). The ”black box” is regarded as a *finite impulse response* (FIR) filter, with its dynamic behavior completely described by the *unit impulse response* **h**

$$r_i = \sum_{l=-\infty}^{\infty} h_l s_{i-l}. \tag{2}$$

Of course, in practice, an approximation of this equation with a finite number of coefficients h_l is used[2]. The unit impulse response (UIR) can be determined from the correlations by inversion of the so-called *Wiener-Hopf equation*[9,10]:

$$\Gamma \mathbf{h} = \mathbf{c}. \tag{3}$$

A frequency response $F(\omega)$ may then be computed as the z-transform $H(z)$ of **h** with argument $z = \exp\{i\omega\Delta t\}$[2,8].

The discussion has so far been limited to the *scalar* frequency response $F(\omega)$ of a one-port. An $m \times m$ transfer *matrix* describing an m-port filter can be estimated with correlation analysis if the signal vector **s** is defined as a suitable combination of the m signal variables $s^{(n)}, n = 1, \ldots, m$, and m impulse response functions $\mathbf{h}^{(n)}$ – one for each response port of the element - are introduced[2]. For acoustic systems, $m = 2$ typically and the variables s, r are the fluctuations of pressure p' and velocity u' or the in- and outgoing Riemann Invariants f and g (see fig. 1).

Reconstruction of a complete transfer matrix in this fashion should provide significant advantages over alternative approaches[2,3]. For example, there is considerable flexibility in the choice of signal shape; superposed sine waves or chirps and in particular bandwidth-limited white noise have produced good results , while step functions – which are resolved well in a fluid dynamics computation only with difficulty – can be avoided[4,6].

SAMPLING AND FREQUENCY INCREMENT

The frequency response $F(\omega)$ of LTI systems is known to be periodic in the range of frequencies $-\pi/\Delta t \leq \omega \leq \pi/\Delta$, and there is symmetry / antisymmetry of the real / imaginary part of the frequency response around the point $\omega = 0$. It follows that for a given sampling interval Δt, only frequencies $f < f_c \equiv 1/2\Delta t$ should be considered[2]. If the time step dt of the CFD model is smaller than the sampling interval Δt, the time series produced by the transient simulation should be lowpass - filtered before carrying out the correlation analysis.

The sample length N, i.e. the duration of the transient simulation, determines the *sampling frequency* (or *frequency resolution*), $\Delta f \equiv 1/N\Delta t$. Formally, with given impulse response **h**, the frequency response of an LTI system can be computed for arbitrary – even complex-valued – frequencies ω with the forward z-transform. This is convenient for dynamic stability analysis of (thermo-)acoustic systems. However, physically it is not sound to evaluate the frequency response for frequencies $f < \Delta f$, i.e. the sampling frequency Δf represents effectively a lower frequency limit f_{min}.

[2]This is the equivalent of the well-known Nyquist criterion.

ORDER AND REALIZABILITY OF THE UNIT IMPULSE RESPONSE

The analysis in the previous section has shown that the order of the unit impulse response filter, i.e. the number of coefficients h_l, does not influence the frequency range and resolution of correlation analysis. However, the *accuracy* of system identification is significantly influenced by the order of the unit response filter. In principle, an infinite number of coefficients, $-\infty < l < +\infty$, is required for complete characterization of an LTI system[8,11]. Of course, in practice only a finite number of coefficients can be handled, and there is a trade-off between numerical and statistical errors (both increase with increasing number of coefficients) and errors due to inadequate resolution of the impulse response.

Experience suggests that with more than, say, 100 coefficients, the determination of auto- and cross-correlations – see eqn. (1) – requires significant computational resources, while the inversion of the Wiener-Hopf equation encounters numerical problems, i.e. the condition number of the autocorrelation matrix Γ becomes very large. On the other hand, the number of coefficients of the unit impulse response required to achieve accurate identification depends strongly on the characteristics of the LTI system under consideration. As it turns out, considerably less than 100 coefficients of the impulse response are often sufficient to characterize multi-ports typically encountered in the analysis of combustion instabilities. Interestingly, it is sometimes advantageous to include coefficients h_l with negative index l. A filter with $h_l \neq 0$ for $l < 0$ is called *non-causal* or *non-realizable*, because the present value r_i of the response is influenced not only by present and previous values of the signal, but also by later ("future") signals s_j, $j > i$, as inspection of eqn. (2) shows. It shall be explained in the following that this surprising finding is not in contradiction with common sense or fundamental principles of physics. The apparent paradox can be resolved by considering how the fundamental fluid-dynamic phenomena are represented in terms of acoustic variables. Also, while a non-causal filter cannot be used for control applications, it causes no essential problems for "off-line" post-processing of time series.

Inverse *z*-Transform

The coefficients of the impulse response h_l may be obtained from the frequency response by the following relation (*inverse z-transform*)[8,11]:

$$h_l = \frac{\Delta t}{2\pi} \int_{-\pi/\omega}^{+\pi/\omega} F(\omega)\, e^{i\omega\Delta t\, l}\, d\omega. \tag{4}$$

This allows to check how accurately a response filter with a finite number of coefficients represents a given frequency response $F(\omega)$: starting from a given frequency response $F(\omega)$, an approximation $\tilde{\mathbf{h}}$ of the unit response with a finite number of coefficients $\tilde{h}_l$, $l = L_{min}, \ldots, L_{max}$ is computed using an approximation of eqn. (4) with a finite number of coefficients $l = L_{min}, \ldots, L_{max}$. The (forward) z-transform of $\tilde{\mathbf{h}}$ then yields an approximate frequency response $\tilde{F}(\omega)$, comparison against the original frequency response $F(\omega)$ then shows to which degree the chosen number and range of coefficients $\tilde{h}_l$ can characterize the LTI system.

System time lags and filter memory

To begin, consider a causal filter with unit response $h_l \neq 0$ only within the range $0 \leq l \leq L_{max}$. The time interval $t_{mem} \equiv \Delta t\, L_{max}$ may be regarded as the *memory* of the filter, because

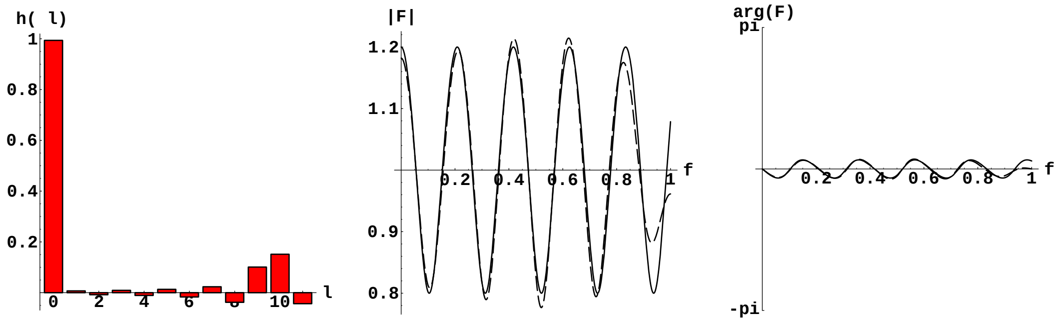

Figure 2: Characterization of the n-τ - model with a UIR filter of order 12. Left: coefficients of the UIR. Middle: absolute value of the frequency response. Right: phase of the frequency response. (——) exact frequency response $F_{n\tau}$ of n-τ model; (— — —) approximation $\tilde{F}$ derived from $h_l, l = 0, \ldots, 11$.

the response at time t_0 is not influenced by signal states prior to $t_0 - t_{mem}$. This observation suggests the following minimal requirement for accurate correlation analysis: L_{max} should be chosen such that the memory t_{mem} of the filter exceeds all delay times or *time lags* τ of the LTI system that is to be identified.

Under certain circumstances, pertinent delay times can be estimated quite easily. For example, if simple propagation of acoustic waves over a distance Δx is the dominant process, then $\tau \approx \Delta x/c$, where c is the speed of sound. On the other hand, if convection (of fuel inhomogeneities, or entropy, e.g.) is significant, then $\tau \approx \Delta x/U$, where U is a representative mean flow velocity.

In situations with complex flow physics, e.g. swirl burners with recirculation zones, more complex relaxation or delay processes may come into play. Then an a-priori estimate of the required L_{max} is not always possible. In this situation, time lags can be deduced from inspection of the cross-correlations between signals and responses – provided that the signals are sufficiently close to white noise in their spectral content!

Example 1: the n-τ Model

A model frequently encountered in studies of combustion instabilities is the so-called *n-τ model* with frequency response

$$F_{n\tau}(\omega) = 1 + ne^{-i\omega\tau}. \quad (5)$$

This simple model is used, e.g., to describe the response of flame heat release to perturbations of velocity u'_u upstream of a premix burner[5]. The time-lag term with *interaction index n* is to represent the relatively slow propagation of flame front perturbations and / or fuel concentration inhomogeneities from the burner mouth to the region of most intense heat release.

If the time lag is a multiple of the sampling interval, $\tau = m\Delta t$ with integer m, it can be shown analytically that the unit impulse response of this LTI system is $h_0 = 1$, $h_m = n$ with all other coefficients being zero. So, for $\tau = \Delta t$ ($m = 1$), a unit impulse response vector of length two is sufficient to completely and exactly (!) characterize this system.

But what if τ is not a multiple of Δt? This question is addressed with numerical analysis,

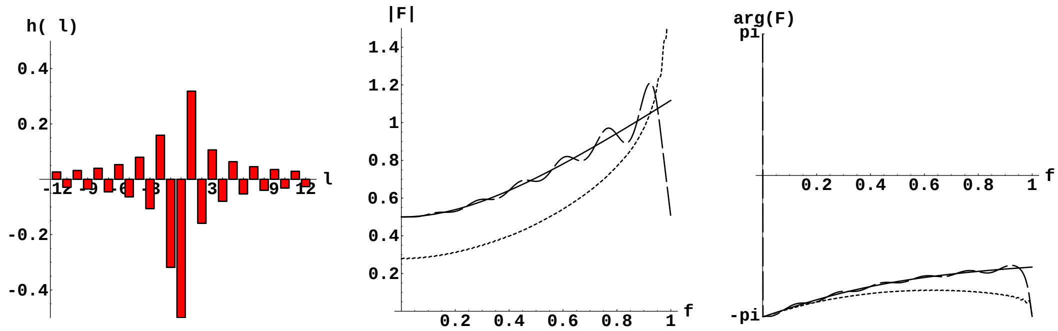

Figure 3: Characterization of the l-ζ - model with causal and non-causal UIR filters. Left: coefficients $l = -12,\ldots,12$ of the UIR. Middle: absolute value of the frequency response. Right: phase of the frequency response. (——) frequency response $F_{l\zeta}$ of the l-ζ model with parameters $\zeta M = 1/2$, $l_{\text{eff}}/c = 1/2\pi$; (— —) $\tilde{F}$ computed from non-causal UIR with 25 coefficients $l = -12,\ldots,12$; (- - -) $\tilde{F}$ computed from causal UIR with 100 coefficients $l = 0,\ldots,99$.

applying as discussed above reverse and forward z-transforms to the frequency response and the approximate UIR $\tilde{\mathbf{h}}$. Results for the case $\tau/\Delta t = 9.6$ and $l = 0,\ldots,11$ are depicted in fig. 2. The unit impulse response (left graph) resembles roughly the expected distribution, with the delayed part of the response appearing primarily in two coefficients $l = 9,10$ bracketing the actual value of the time lag. With just one dozen coefficients for the UIR, the approximate frequency response $\tilde{F}(\omega)$ is quite similar to the original, except near the Nyquist frequency. As one would expect, the agreement between $F_{n\tau}(\omega)$ and $\tilde{F}(\omega)$ improves with increasing L_{max}. However, deviations very near the Nyquist frequency remain no matter how many coefficients are used, c.f. the well-known Gibbs-Phenomenon. Therefore it is recommended to choose the sample interval Δt such that the corresponding Nyquist frequency f_c is 20 - 50 % higher, say, than the maximum frequency of interest. Then the deviations of frequency responses from the correct value near the Nyquist frequency can not influence the results of network analyses with the reconstructed transfer matrix.

Example 2: the l-ζ Model (Compact Element with Loss)

It has been suggested that at sufficiently low frequency, the coupling between fluctuations of pressure and velocity across a premix swirl burner may be approximated as

$$\begin{pmatrix} \left.\frac{p'}{\rho c}\right|_d \\ u'_d \end{pmatrix} = \begin{pmatrix} 1 & -\zeta M - i\omega\frac{l_{\text{eff}}}{c} \\ 0 & \alpha \end{pmatrix} \begin{pmatrix} \left.\frac{p'}{\rho c}\right|_u \\ u'_u \end{pmatrix}, \tag{6}$$

where α would be an area ratio between up- and downstream cross-sections, l_{eff} an *effective* or *reduced length* and ζ a pressure loss coefficient[12]. Indeed, for compact flow elements with a contraction of cross-sectional area along the flow path, this approximate description seems adequate for a wide range of conditions[4]. Here we are concerned primarily with the upper right coefficient, a plot of the frequency response $F_{l\zeta}(\omega) = -\zeta M - i\omega l_{\text{eff}}/c$ is shown in fig. 3. It turns out that this coefficient of the transfer matrix cannot be represented well by a causal impulse response filter h_l, $l = 0,\ldots,L_{max}$ even if a very large number of coefficients is used.

However, the slope of the phase of the frequency response is positive, as would be the case also for a time-lag system $F(\omega) \sim \exp(-i\omega\tau)$ with *negative* time lag. This suggests that it should be possible to represent an l-ζ system with a non-causal unit response. This is indeed the case, the approximate frequency response $\tilde{F}$ obtained with $l = -9, \ldots, 9$ is shown in figure 3.

Causality and the proper Choice of Variables

The failure of a causal response filter to characterize the l-ζ - model is of course not due to any effects in acoustic wave transmission and reflection which violate causality. Instead, the root cause of the problem is that the variables used, i.e. primitive acoustic variables p' and u' with the upstream location taken as the "signal" and the downstream location as the "response", respectively, do not properly represent the underlying cause-and-effect relationships imposed by fluid mechanics and acoustics.

This is best explained with a simpler example, i.e. propagation of plane waves between two locations "u" (upstream) and "d" (downstream) over a distance Δx with transfer matrix

$$\begin{pmatrix} \left.\frac{p'_d}{\rho c}\right|_d \\ u'_d \end{pmatrix} = \begin{pmatrix} \cos(k\Delta x) & -i\sin(k\Delta x) \\ -i\sin(k\Delta x) & \cos(k\Delta x) \end{pmatrix} \begin{pmatrix} \left.\frac{p'_u}{\rho c}\right|_u \\ u'_u \end{pmatrix}. \tag{7}$$

The cause-and-effect relationships in this example are very simple: a downstream travelling wave f_u at the upstream location should be considered as a signal, which leads to a response f_d at the downstream location after a time $\Delta x/c$, while a wave g_d coming in from the downstream side has a response g_u. Obviously, the notation in terms of primitive variables p' and u' used in (7) does not really reflect these interrelations; a more adequate choice of variables is obviously,

$$\begin{pmatrix} f_d \\ g_u \end{pmatrix} = \begin{pmatrix} e^{-i\omega\Delta x/c} & 0 \\ 0 & e^{-i\omega\Delta x/c} \end{pmatrix} \begin{pmatrix} f_u \\ g_d \end{pmatrix}, \tag{8}$$

which makes use of the *scattering matrix*. Note that the two non-zero coefficients of the scattering matrix have a negatively sloped phase w.r.t. frequency, corresponding to a positive time lag.

Correlation analysis with CFD data for plane wave propagation shows indeed that with a causal UIR ($l = 0, \ldots, L_{max}$) all matrix coefficients are reproduced correctly only if the analysis is formulated in terms of the scattering matrix with f_u, g_d as "signals" and f_d, g_u as "responses". Conversely, if a non-causal UIR is used, it does not make a significant difference whether Riemann invariants f, g or primitive variables p', u' are used and which are considered to be "signals" and "responses", respectively. Due to lack of space, these results can not be presented and discussed in more detail in this paper.

These findings suggest that it should be possible to avoid the use of non-causal filters if variables can be identified, which respect the causal relationships of the problem at hand. However, it turns out that this is not feasible even for a system as simple as the l-ζ model. The following explanation is offered for this result: the loss term ζM expresses the fact that a change in mass flow rate through the element results in a change in pressure drop, hence u' should be regarded as the "cause" and p' as the "effect". On the other hand, the term with effective length l_{eff} represents the inertia of the fluid between "u" and "d", such that a change in pressure difference $p'_u - p'_d$ will lead to a gradual change in velocity u' – the roles of cause and effect are exchanged! Indeed, it has been observed that it is in general not possible to accurately characterize the l-ζ - model with a causal response filter, no matter which notation is used.

CONCLUSIONS

As a result of the present study, the following recommendations for accurate identification of acoustic multi-ports from transient CFD simulation can be formulated : a) identify a maximum frequency of interest f_{max}, then low-pass filter the raw time series data obtained from the simulation with cut-off f_{max}. b) choose the time increment Δt as a multiple of the simulation time step dt such that the corresponding Nyquist frequency is 20 to 50 percent higher than f_{max}. c) make sure that the simulation has been carried on for a sufficiently long time such that the lower frequency limit $f_{min} = 1/N\Delta t$ is acceptable. d) identify possible time-lag mechanisms and estimate a maximum time lag τ. If the spectral content of the signals is close to white noise, then time lags may be inferred from cross-correlations between signals and responses. e) carry out correlation analysis with a non-causal unit impulse response $h_l, l = -L, \ldots, L$ with L chosen such that $L\Delta T > \tau$. f) if a formulation for acoustic wave reflection and transmission at the multi-port can be found, which properly reflects the causal interrelations between ”signals” and ”responses”, then a causal UIR $h_l, l = 0, \ldots, L$ may be used to reduce computational effort and numerical error.

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