INFLUENCE OF BOUNDARY REFLECTION COEFFICIENT ON THE SYSTEM IDENTIFIABILITY OF ACOUSTIC TWO-PORTS

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Abstract

For the purpose of analysis or control of complex (thermo-)acoustic systems, so-called network models are popular. With this approach, individual system elements are represented as two-ports, described mathematically by their respective transfer matrices. Tools from system identification can be used to estimate transfer matrices from transient CFD data. As the application of system identification is relatively new in the acoustics context, many possible influences on the accuracy and robustness of the identification are still unclear.

The present study investigates how strongly the results of the identification procedure are influenced by the boundary conditions of the system model, particularly the acoustic reflection coefficient of the boundaries. A real time network model is used for this benchmark study. Such a formulation is not only numerically very efficient, it also allows to determine quantitatively an *estimation error* by comparing identification results against the transfer matrices from which the real time model is built. Most important, arbitrary values for the reflection coefficient at the system boundaries may be prescribed. This would not be possible in a CFD model.

It is observed that the estimation error increases significantly with the absolute magnitude of the reflection coefficient. Above a certain threshold value, the identification procedure fails completely. The reason for this dependence is that with increasing reflection coefficient resonant amplification of the excitation signal becomes more pronounced, leading to linearly dependent signals and an ill-conditioned auto-correlation matrix. It must be conclude that CFD formulations with non-reflecting boundary conditions should be used for system identification of acoustic two-ports whenever possible.

INTRODUCTION

So-called *network models* are frequently used for the analysis of sound propagation in ventilation or exhaust duct systems[1]. They are also popular tools for the analysis of self-excited combustion instabilities, e.g. in gas turbines. The building blocks of these models are *two-ports*. Each two-port corresponds to a particular component or element of the (duct) system under investigation. Mathematically, two-ports can be represented in the frequency domain by *transfer matrices* T, which may be written

$$\begin{pmatrix} f_d \\ g_u \end{pmatrix} \equiv T(\omega) \begin{pmatrix} f_u \\ g_d \end{pmatrix}. \tag{1}$$

Here f and g are the amplitudes of down- and upstream traveling waves, respectively, at locations u and d at the up- and downstream ports of the element. Equivalent formulations based on

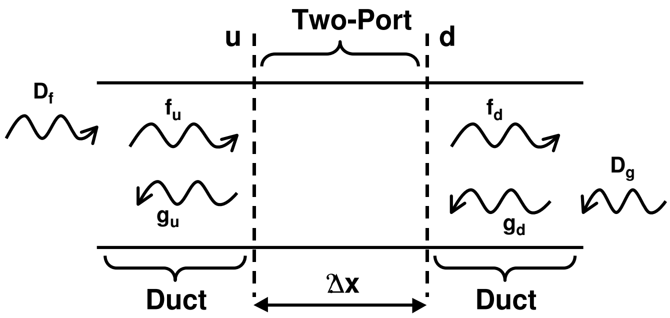

*Figure 1: Schematic of CFD / real time network setup for identification of an acoustic element (*boundary – duct – element-to-be-identified – duct – boundary*). The two-port to be identified is part of the computational domain / network model between monitor locations u and d. Excitation of the system at the boundaries (D_f and D_g) generates signals $s_1 = f_u$ and $s_2 = g_d$ impinging on and responses $r_1 = f_d$ and $r_2 = g_u$ emanating from the two-port.*

acoustic pressure fluctuations p' and velocity fluctuations u' are also used, with the relations

$$f = \frac{1}{2}\left(\frac{p'}{\rho c} + u'\right), \; g = \frac{1}{2}\left(\frac{p'}{\rho c} - u'\right), \tag{2}$$

between f, g and the primitive acoustic variables p', u'. Mean flow density and speed of sound are denoted as ρ and c.

The coefficients T_{ij} of the transfer matrices may be determined by analytical means for simple acoustic elements. Experimental characterization of two-ports is also possible, but tedious and time-consuming[2]. More recently it has been proposed to *estimate* or *reconstruct* acoustic transfer matrices from CFD data with methods developed in the context of signal processing and in particular *system identification*[3,4]. The reader should be aware that the established terminology is somewhat misleading here: in the present context, "system identification" tools are used to characterize a particular acoustic *element*, which is part of a larger system (see Fig. 1). The transient behavior of the system under external, broad-band excitation is modelled to generate time series, which are interpreted as signals impinging on and responses emitted from of the two-port under investigation. These time-series are then post-processed to obtain the transfer matrix for a range of frequencies from *correlation analysis*.

This combination of transient CFD simulation with a post-processor for the estimation of acoustic transfer matrices was introduced by Polifke et al.[4] with an application to a heat source with time lag and validated against analytical results. Gentemann et al.[5] reconstructed and validated successfully the transfer matrix of a sudden change of cross sectional area between two ducts. Issues of frequency resolution, optimal choice of signal variables, optimal order of impulse response filters for system identification, and the relation to filter realizability have been discussed by Polifke and Gentemann[6]. Most recently the method was applied by Gentemann at al.[7] to a generic premix burner and validated against experiment. An introduction to the method and a brief review of the results accomplished to date has been published recently[8].

As the application of system identification is relatively new in the acoustics context, many possible influences on the accuracy and robustness of the identification of acoustic elements remain unclear. For example, Polifke and Wall have drawn attention to the acoustic reflection coefficient R of the boundary conditions of the CFD model[9]. It is well known that broad-band excitation signals in conjunction with high boundary reflection coefficients will result in

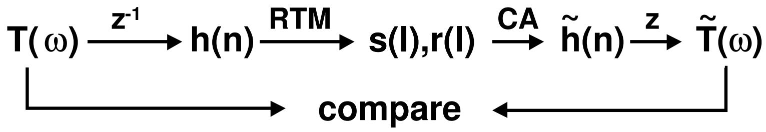

Figure 2: Flow chart for validation of system identification with real time model (RTM).

strong resonant amplification at selected frequencies. This in turn presents a serious problem for the identification procedure, as resonant amplification can result in linearly dependent input signals $s_1(= f_u)$ and $s_2(= g_d)$ at the monitor planes – resulting in a poorly or ill-conditioned autocorrelation matrix – and inadequate signal-to-noise ratios for non-resonant frequencies. In short, large reflection coefficients $|R| \to 1$ will make it difficult (if not impossible) to properly reconstruct the transfer matrix *.

Clearly, it would be preferable to employ a CFD scheme with low acoustic reflection coefficient $|R| \to 0$ at the boundaries. However, even characteristics-based boundary conditions [10] present a high reflection coefficient to low frequency acoustic waves[9,11]. Polifke and Wall have proposed modified characteristics-based boundary conditions with "masking" of outgoing waves in the so-called LODI-relations[9–11]. However, the implementation of these boundary conditions in a CFD code is not trivial, and therefore it would be important to known just how strongly the quality of identification results depends on the magnitude of the reflection coefficient. In other words: is the implementation of non-reflecting boundary conditions in a CFD code worth the effort? The present paper is concerned with an investigation of this question.

For such a study, a CFD model of the acoustic system (and the two-port to be identified) is not the appropriate basis. Firstly, CFD implies long compute times. Secondly, in a CFD simulation numerical errors or inadequate physical models may also influence the accuracy of the identification results. Similarly, the true transfer matrix of a particular element – say a sudden change in cross-sectional area in a duct or a flame – is not known exactly a priori, so a rigorous validation of identification results is difficult. Thirdly – and most importantly – the acoustic reflection coefficient can not be altered freely in a CFD code.

To circumvent these problems, a *real time network model* based on transfer matrices – which are assumed to be known a priori – emulates the CFD simulation and the identification procedure. This model formulation is discussed in detail in the next section. It is emphasized that the boundary conditions in the real-time network model can be adjusted freely (see below), and thereby the impact of the reflection coefficient on the quality of matrix reconstruction can be investigated quantitatively. Three test cases are considered: plane wave propagation in a duct, wave transmission and reflection at a change in cross-sectional area between two ducts, and at a premix flame.

REAL TIME NETWORK MODEL

The following issues are discussed in this section: a) overall scheme for validation of the system identification with variable reflection coefficient b) formulation for a two-port and a complete system model in the time domain (real time model, RTM) c) structure of the RTMs used in this study.

*Indeed, it can be shown analytically that for $|R| = 1$ with complete reflection on both system boundaries, all signals are linearly dependent, which makes system identification of two-ports definitely impossible.

Validation of system identification

The validation scheme for system identification based on a real time system model is outlined in figure 2 and comprises the following steps:

1. for the acoustic two-port to be identified, a time domain representation is constructed by the inverse z-transform[8,12] of the transfer matrix coefficients:
$$h_{ij}(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} T_{ij}(\omega)\, e^{i\omega n} d\omega. \tag{3}$$
2. a real time network model (RTM) generates the transient system response to excitation and produces time series $s_i(l)$ and $r_i(l)$ of the acoustic element to be identified. Here $i = 1,2$ for the four poles of each element.
3. from the time series $s_i(l)$ and $r_i(l)$, the unit impulse responses $\tilde{h}_{ij}$ are estimated with correlation analysis (CA). Then the transfer matrix $\tilde{T}$ computed by forward z-transform. These steps employ exactly the same algorithms as used for time series produced with ”real” CFD data[4,8].

Eventually, the original transfer matrix T and the estimated counter part $\tilde{T}$are compared against each other.

Formulation of the real-time model

The real time representation of a two-port can be based on the unit impulse responses h_{ij}, which in turn are determined from the transfer matrix, see Eqn. (3). In particular,

$$r_j(l) = \sum_{i=1}^{2}\sum_{n} h_{ij}(n) s_i(l-n), \tag{4}$$

where s_i and r_i, $i = 1,2$ denote the incoming signals and the outgoing responses of the two-port, see Fig. 3. Individual two-ports are then connected by passing input and output signals from one two-port to the next to build an (open loop) system model. Finally two elements, representing the reflective boundaries, are added to close the system, see figure 4. The reflection coefficients are defined as

$$r_u = \left|\frac{f}{g}\right| e^{i\theta}, \quad r_d = \left|\frac{g}{f}\right| e^{i\theta}, \tag{5}$$

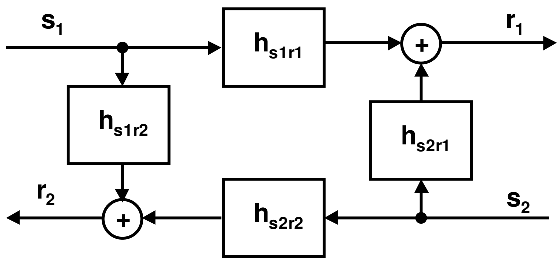

Figure 3: Building block of a generic Two-Port Element.

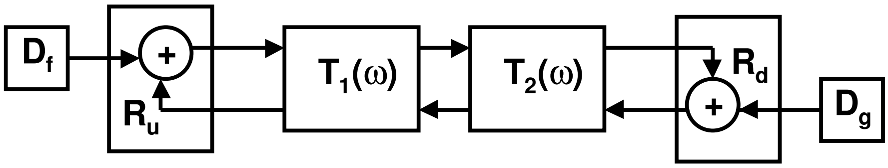

Figure 4: Closed loop system model consisting of two acoustic elements 1 *and* 2.

with θ set to $-\pi$ for this study. Such a formulation is easily implemented in SIMULINK[13]. Once external forcing signals are provided, the dynamics of the system can be simulated. Similar approaches to real time system modeling were proposed by Schuermans et al.[14].

System models used in this study

Three different system models were constructed to validate the transfer matrix estimation procedure for plane wave propagation in a duct, across a sudden change of cross sectional area, and a premix flame, respectively. The most complicated system is the one including the premix flame. This model, which corresponds to a simple combustor rig, see Fig. 5, is explained in detail here. The other system models are simpler sub-sets of this ”combustor model” and not discussed further.

The n-τ Model is used to model the heat release in the flame ($T_{n\tau}$)[15]. At ”monitor locations” before and after this central element, time series of signals and responses are extracted from the model for post-processing (s and r boxes in the figure). Upstream of the flame element, an area change, indicated by T_{AC} represents the ”dump plane” of the combustor[5]. Elements which model plane wave propagation (T_{PW}) are added on both the upstream and the downstream side.

The acoustic conditions provide the opportunity to specify the desired reflection coefficient (R) and the external driving signal for the f and g waves, as indicated by the symbol D. In this study, only broad-band ”white noise” signals were used. Note that the signal s and responses r at the ports of the element to be identified are not equal to the driving signals!

The other two, simpler system models used consist of the following components: *boundary – duct– area change – duct – boundary* and *boundary – duct– duct – duct – boundary*. The element to be identified was always the central one, i.e the *area change* and the *duct*, respectively.

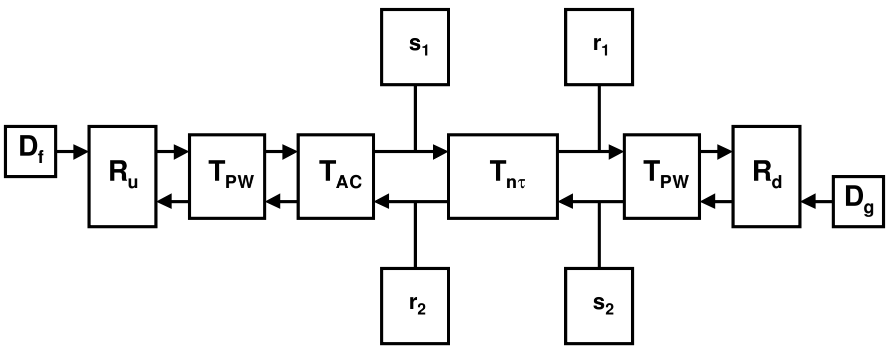

Figure 5: System model for a generic combustor with n-τ heat source ($T_{n\tau}$). T_{PW}: Plane wave propagation (time lag). T_{AC}: area change

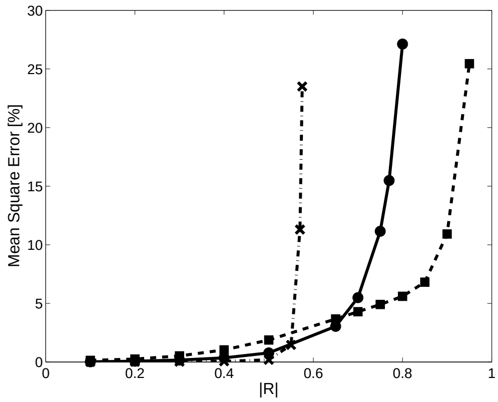

Figure 6: Estimation error of identification vs. magnitude of reflection coefficient.
■: Plane wave propagation, ●: area change, $\times$: $n-\tau$ Model.

RESULTS

The influence of the boundary reflection coefficient on the identification of acoustic two-ports is investigated, using the real time system models described in the previous section. The reflection coefficient $|R|$ – assumed equal on both boundaries and not depending on frequency – has been varied between zero and unity.

For quantitative comparison between the original transfer matrix T and the estimated counterpart $\tilde{T}$ (see Fig. 2) a normalized *estimation error* is defined as

$$\varepsilon(|R|) = \sum_{i,j}\sum_{\omega} \frac{|T_{ij}(\omega) - \tilde{T}_{ij}(\omega)|^2}{|\tilde{T}_{ij}(\omega)|^2}. \tag{6}$$

For the three different system models described in the previous section, the dependence of the estimation error ε on the magnitude of the reflection coefficient is shown in figure 6. Squares denote the result for the plane wave model, the circles represent the result for the area change model, and the crosses stand for the $n-\tau$ flame model. In general, a significant dependence of the estimation error ε on $|R|$ is observed. For $|R| \leq 0.55$, $\varepsilon < 2\%$ (all models). The estimation error increases much stronger with $|R|$ for the area change model than it does for plane wave propagation. For the $n-\tau$ model, this effect is the strongest. For all models, a maximum reflection coefficient $|R|_{max}$ can be observed. This quantity is defined as follows: for $|R|$ below this maximum value, a qualitative agreement between the given and the estimated transfer matrix can be observed over the whole frequency range (not shown). Above $|R|_{max}$, the estimated transfer matrix results in seemingly arbitrary values, i.e. identification fails completely. For the system and element models investigated in this study, $|R|_{max} = 0.99$ for the plane wave model, $|R|_{max} \approx 0.85$ for the area change, and $|R|_{max} \approx 0.6$ for the n-τ model. One may state that for more complex elements, the magnitude of the reflection coefficient needs to be smaller to ensure identifiability of the element.

It has been argued above that resonant amplification resulting in linearly dependent signals and ill-conditioned system matrices can account for the influence of the reflection coefficient on the estimation error. The value of $|R|$ influences indeed the magnitude of the resonance peaks. Figure 7 (left) shows the power spectral density (PSD) of the signal f_u (normalized with the peak magnitude at $|R| = 0$) for the ”area change” and $|R| = 0.93$. It is observed that resonance

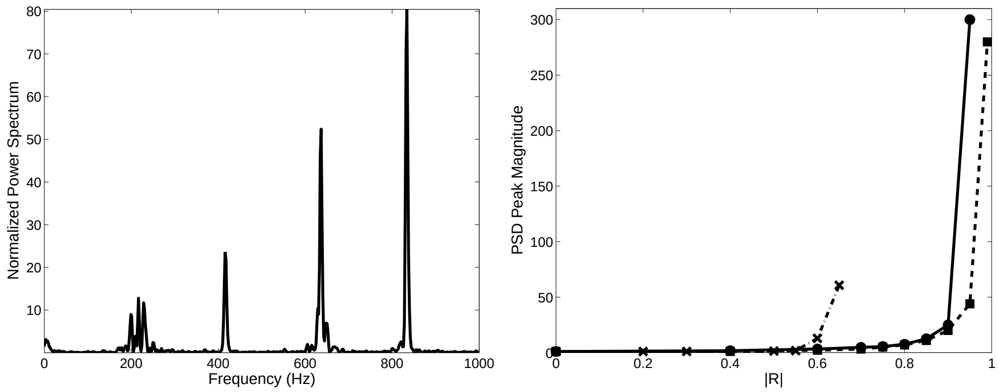

Figure 7: Normalized power spectral density (PSD) of signal f_u for the "area change". Left: $|R| = 0.93$. *Right: normalized maximum peak magnitude of PSD of f_u as a function of $|R|$.*

peaks dominate the power spectral density distribution, no visible trace remains of the white noise forcing signal. A maximum peak magnitude of about 80 is observed. Note that for $|R| < 0.6$, resonant peaks are hardly observed (not shown).

Figure 7 (right) shows the normalized maximum peak magnitude of the power spectral density of f_u as a function of $|R|$. The increase of the maximum peak magnitudes in the PSD is observed at roughly the same $|R|$ as the identification boundary (figure 6). This strongly suggests that a correlation between the diminished identification quality and the observed resonance exists. However, a general rule, which would link the height of the resonance peaks to the distance from the identification boundary for a given model, could not be established.

A similar behavior is found in terms of the correlation between the signals. The correlation coefficient C is defined as

$$C = \frac{Cov(s_1, s_2)}{\sqrt{Var(s_1)Var(s_2)}}, \tag{7}$$

where $Cov(s_1, s_2)$ is the covariance between s_1 and s_2 and $Var(s_1)$ the variance of s_1. Increasing $|R|$ over $|R|_{max}$ results in the correlation coefficient C between signal one and two to approach unity very quickly. For all models it is observed that C increases with the maximum peak magnitude of the resonance peaks. Thus, the reflection coefficient is directly proportional to the identifiability of acoustic elements.

CONCLUSION

The influence of boundary condition reflection coefficient on the identifiability of acoustic two-ports is investigated. This was done by means of a real time system models based on the unit impulse response filters and implemented in SIMULINK. Three different two-ports, representing plane wave propagation in a duct, plane wave transmission and reflection at a sudden change of cross sectional area, and the n-τ flame model have been investigated.

It is found that the identifiability of a two-port depends strongly on the type of two-port to be identified, the model system setup used for the identification and in particular on the boundary reflection coefficient R. In general, accuracy decreases with increasing values of $|R|$, and for all two-ports investigated, a certain reflection coefficient $|R|_{max}$ is observed above which identification of the system is impossible. More complex elements tend to require a lower boundary reflection coefficient for accurate identification. The reason for the observed effects seems to be acoustic resonance of system models with large boundary reflection coefficients, which results in strongly correlated signals and an ill-conditioned auto-correlation matrix.

It must be conclude that CFD formulations with non-reflecting boundary conditions should be used for system identification of acoustic two-ports whenever possible.

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